

Homework 2 - Approximate representation theory (pmath 945)

Please hand in homework by email on Wednesday, September 25th.

(1) Let

$$\mathcal{A} = \langle x, y : xy - yx - 1 \rangle$$

be the Weyl algebra.

(a) Show that the C^* -enveloping algebra of \mathcal{A} is zero.

(b) Show that \mathcal{A} has a representation on the vector space $\mathbb{C}[z]$ of polynomials in one variable, where x acts as differentiation $\frac{d}{dz}$ and y acts as multiplication M_z by z . Conclude that $\mathcal{A} \neq 0$.

(2) Let $\|\cdot\|$ be a unitarily invariant norm on $M_n\mathbb{C}$. Show that

(a) $\|a\| \leq \|b\|$ for all $a, b \in M_n\mathbb{C}$ with $0 \leq a \leq b$.

(b) $\|abc\| \leq \|a\|_{op}\|b\|\|c\|_{op}$ for all $a, b, c \in M_n\mathbb{C}$.

(3) An exercise from this week's lecture / lecture notes of your choice.