Homework 3 - Approximate representation theory (pmath 945)

Please hand in homework by email on Wednesday, October 9th.

(1) Let $\mathcal{A} = \mathbb{C}^*\langle u : u^*u = 1\rangle$. Show that $\mathcal{A}$ is stable with respect to $\mathcal{F} = \{(B(H), \|\cdot\|_{op}) : H \text{ Hilbert space}\}$.

(2) Let $\mathcal{A} = \mathbb{C}^*\langle u, v : u^2 = v^2 = 1, u^* = u, v^* = v, uv = vu\rangle$. Show that there is a function $f : [0, C] \to \mathbb{R}_{\geq 0}$ with $C > 0$ and $\lim_{\epsilon\to 0} f(\epsilon) = 0$ such that if $\phi : \mathbb{C}^*\langle u, v\rangle \to M_n\mathbb{C}$ is an ϵ-representation, $\epsilon \leq C$, with respect to a unitarily invariant norm $\|\cdot\|$ with $\phi(u)$ and $\phi(v)$ unitary, then there is a representation $\psi : \mathcal{A} \to M_n\mathbb{C}$ such that $\|\psi(u) - \phi(u)\| \leq f(\epsilon)$ and $\|\psi(v) - \phi(v)\| \leq f(\epsilon)$.